PRICE SIGNALS IN A UNIT COMMITMENT-DRIVEN MARKET

Submitted to
The Engineering Honors Committee
119 Hitchcock Hall
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Friday, May 20, 2011

ABSTRACT

Electric power systems are made up of many generators, each with varying characteristics, including: minimum and maximum generating capacities, start up costs, and costs to run. An independent system operator uses unit-commitment models to determine which generating units should be in use at a given time in order to maximize the social welfare of electricity use. Because of the size and complexity of these models and the limited time in which to solve them, they are typically solved only to near-optimality. Although these near-optimal solutions are similar in terms of overall welfare, energy prices and profits to individual generators can vary significantly between these solutions. A unit commitment model was run for one year using varying demand elasticities value and optimality gaps. The results show that generator profits do not even out over the year and generators can have significant profit differences. Solving closer to optimality does not reduce this effect. Average price differences for the year are small, but there are a significant number of hours during which the price difference between optimal and near-optimal is significant.

ACKNOWLEDGEMENTS

I would like to thank my thesis advisor, Dr. Ramteen Sioshansi, for his guidance and assistance throughout this project. He went above and beyond to help me through learning new computer programs. Additionally I would like to thank my Operations Research professors, Dr. Sioshansi and Dr. Marc Posner, for teaching me the fundamentals of linear and nonlinear programming. Furthermore, I would like to thank all of the teachers within the Integrated Systems Engineering department who have contributed to my education. I would also like to acknowledge my advisor, Pam Hussen, for all of her help and continued support throughout my academic career. Finally, I would like to thank my parents for their emotional and fiscal support.

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1.0 Introduction

In recent years, electric power systems have moved toward a structure in which electricity generation, transmission, and distribution assets are owned and operated by different entities. An independent system operator (ISO) is a separate entity, independent of any of the stakeholders, that determines generation and transmission schedules that maximize the social value of the power system assets (Sioshansi et al. 2008). The ISO is generally charged with social welfare maximization as its objective. Social welfare is defined as the sum of consumer and producer surplus.

In order to maximize social welfare the ISO must solve a unit commitment that determines which generating units should be used at any given time. This problem is a mixed-integer program (MIP), which uses binary variables to represent the on/off nature of generator operations. For the power system to work, the total generation must equal the demand at every point in time. Since the demand is constantly changing there must also be a reserve of excess generating capacity, which is ready to generate on short notice (Borenstein et al. 2002). Schedules for generators are typically completed a day ahead. Since the ISO has a limited timeframe in which to solve its optimization models and determine schedules for the following day, the unit commitment problem is typically not solved to complete optimality and a near-optimal solution is used instead. Different levels of near-optimal solutions exist. The optimality gap or optimality tolerance measures how close the near-optimal solution is to the optimal solution. This measurement is the percent within the optimal value that the final solution is guaranteed to be within. For example, a near optimal solution of 1e-4 means the final integer solution is guaranteed to be within

0.01% of the optimal value. The area around the optimal solution is very flat. Thus these near-optimal solutions have very little difference in terms of social welfare, but can vary significantly in terms of energy prices and profits of individual generators. Previous work has focused on modeling only a short amount of time. This project ran a unit commitment model for an entire year. The purpose of modeling an entire year is to see if the variability in individual days balances over a year or if they build throughout the year. Determining if certain types of generators are more prone to variability is another goal. The generators have a wide range of costs and operating constraints. In order to examine the difference between solving to a near-optimal and an optimal solution, the model is optimized using different termination criteria. Energy price and generator profit differences between these different cases are compared.

This research will also examine what effect price-responsive electricity demand would have on prices and profits associated with near-optimal unit commitment solutions. For many years economists have claimed benefits of using markets to send price signals to market participants and have justified these claims with simple models. Yet no one has yet shown whether the prices generated by a unit commitment-driven market provide useful signals. This project will work toward a higher goal of determining if restructured markets send these valuable price signals. Prices signal two sides of the market: investors and consumers. Prices help investors determine whether to build new generation. The generator profit analysis explores this aspect of price signals. The other part of price signals is sending informative signs to the consumer. Varying elasticity values and looking at prices touches on this aspect.

1.1 Literature Review

Unit commitment is defined as a decision indicating what generating units are to be in use at each point during a scheduling horizon. A related activity is economic dispatch, which is determining the allocation of the demand for power among generating units in operation at any moment in time (Muckstadt et al. 1976). Muckstadt et al. present a unit commitment model, which shares many key aspects of the model that is used in this experiment. They also demonstrate that there can exist many near-optimal solutions to a unit commitment problem (Muckstadt et al. 1976).

Johnson et al. (1997) examine the implications of these many near- optimal solutions in a competitive market setting. They use a test system with a 168-hour planning horizon to demonstrate that while near-optimal solutions are very similar in terms of overall welfare, they can yield very different profits to individual generators (Johnson et al. 1997). They show that solutions with under one-tenth of a percent variability in the total system cost can result in aggregate resource profits varying up to six percent. They also demonstrate strong dependencies between decisions in successive hours. Overall they support a more decentralized approach to unit commitment problems (Johnson et al. 1997).

Guan et al. (2003) show that using heuristics to solve a unit commitment problem can cause systematic biases against specific market participants. The paper promotes a solution method that combines the general MIP method with the Lagrangian Relaxation framework to take advantage of both approaches (Guan et al. 2003).

Sioshansi et al. (2008) demonstrate that both Lagrangian relaxation and MIP solution methods will yield the same equity issues that Johnson et al. (1997) discuss, when the problems are not solved to complete optimality. They also demonstrate that different near-optimal solutions can yield very different energy prices, depending on which generators are online and marginal in a particular hour.

2.0 Project Objectives

The objectives of this project are:

1.0 Determine the amount prices fluctuate when unit commitment models are not able to be solved to complete optimality.

1.1 Examine whether demand elasticity reduces price fluctuations.

2.0 Determine if there are systematic fluctuations in the profits of individual generators.

2.1 Establish if the profit fluctuations balance over one year.

The objectives of this project relate to the overall purpose of determining if restructured energy markets send valuable price signals, by determining if certain characteristics of generators affect their profit differences. It also relates to the overall purpose by examining whether demand elasticity can reduce price and profit fluctuations.

3.0 Project Approach

In order to complete the project objectives, a unit commitment model was developed and run, using a year of power system data from the ISO New England electricity market. To accomplish the first objective locational marginal prices (LMPs) from the near-optimal and optimal solutions are examined. Throughout the project three optimality gap values are considered: 1e-2, 1e-4 and 1e-6. The LMP is the marginal value of an additional unit of electricity (being consumed or produced) at each location that is modeled. How the LMP is computed is explained in section 5.0. The difference in price in each hour of each day is examined. General statistics about the overall differences for the year are examined as well as examining specific outliers and trends. Objective 1.1 is accomplished by running the model at a -0.1, -0.2, and -0.3 elasticity values to both near- and complete-optimality. These runs are analyzed similarly to the cases without demand elasticity.

The second objective is accomplished by looking at the differences in each individual generator's daily profit. The daily profit differences will be looked at as well as total annual profit differences. Generators that have the most significant difference in yearly profits will be examined closely. Characteristics of the generators will be examined to see if any trends exist in the types of generators which experience the most significant difference in profits.

The following are the specific phases of the project.

I. Learning Phase: Learn how to program optimization models in AMPL and conduct literature review on pricing in markets with nonconvexities.

II. Data Collection: Gather demand data for 2005 from ISO New England, which are available from the ISO New England website. Retail electricity price data for the same time period will also be collected from the U.S. Department of Energy's Energy Information Administration.

III. Model Formulation Phase: Formulate the unit commitment model used for the analysis. This model was presented and approved by the faculty advisor before moving to the next phase.

IV. AMPL Modeling Phase: Build an AMPL model that runs for an entire year, solving for a 48 hour time period and keeping the solution for the first 24 hours and then rolling forward. The model should be able to run using price-elastic bids. A variety of constraints must also be considered in building the model. These constraints are detailed in section 4.

V. Model Run Phase: The various cases are run using optimization software. The cases run use either actual 2005 load data, or a price-elastic demand function with a demand elasticity of -0.1, -0.2, or -0.3. The models are run with an optimality gap of either 1e-2, 1e-4, 1e-6, or 0. Thus a total of 16 different cases with demand and optimality criteria are run. The computer run phase will take approximately two weeks.

VI. Output Analysis: Output from the trials is analyzed using Microsoft Excel. The output focused on was generator profits and LMPs. Specifically, generators that have large profit differences between cases will be examined.

VII. Report Phase: An oral defense of a draft thesis before a faculty committee was completed on May 9th 2011. A report documenting the research and findings was written and submitted to the Knowledge Bank.

The expected timeline of this project is as follows: Phases I, II and III will be completed by the end of the 2010 autumn quarter, phase IV and V will be completed by the end of winter quarter 2011, finally phase VI and VII will be completed by the eighth week of spring quarter 2011. For a more detailed project timeline see Appendix A for a project Gantt chart.

4.0 Model

The model used is a basic unit commitment model with ramping constraints, minimum-up and -down times, marginal, fixed, and startup costs and reserve requirements. The model is intended to be solved as a MIP with a rolling 48-hour planning horizon. The model uses 2005 ISO New England data and includes eight demand zones, 276 generators, and six transmission lines. The demand data used is hourly demand data for each day in 2005. The generator characteristic and power flow data are based off of a single day's data. Therefore a feasible solution is not able to be found without relaxing the ramping constraints on eight days through over-generation and under-generation variables. In order to limit the amount of time this was done a high cost was associated with over-generation or under-generation. Since the variables representing over-generation and under-generation are only used for feasible running of the model they are not presented in the model formulation in this section of the report. The full AMPL program can be viewed in Appendix B.

4.1 Variables and Parameters

There are a variety of variables and parameters used to build the model. The parameters relate to network flow (transmission lines), generator capacity, time, demand and cost. There are both binary and continuous variables used in the model. Most variables relate to the generators and their activity.

The transmission parameters that needed to be taken into account were the limit on the six transmission lines and the amount of input and output on each line from the generators and demand in the eight zones.

The generators each had minimum and maximum generating capacities as well as limits on the amount of increase and decrease between each hour (ramping constraints). Each generator remains on/off for a minimum amount of time once entering a startup/shutdown. There are certain times that each specific generator must run.

Costs are accrued when a generator is started up and a marginal cost occurs based on the amount of electricity produced. Fixed costs for each generator are also present in the model. The parameters can be seen in the table 4.1 below.

Parameter	Meaning
c_i^f	Fixed cost for generator i
c_i^m	Marginal cost for generator i
c_i^{start}	Startup cost for generator i
$mincap_i$	Minimum load of generator i
$maxcap_i$	Maximum load of generator i
$maxdec_i$	Maximum decrease of generator i
$maxinc_i$	Maximum increase of generator i
$minon_i$	Minimum on time of generator i
$minoff_i$	Minimum off time of generator i
$mustrun_{i,t}$	Binary; 1 when generator i must be on in hour t
$L_{c,t}$	Interface limit on each line in each hour
$ptdf_{c,n}$	Power transfer distribution factor on each line for each node
$init_i$	Initial number of hours generator i has been online (negative means offline)
s_i^{hour}	Hour in which a generator starts up due to a startup in a previous period

Table 4.1: Definition of parameters

The binary variables determine when the generators are in use, when they are started up and when the generators shutdown. A set of continuous variables is used to determine the amount of generation from each generator.

The reserve requirements used in this model require that within 10 minutes there is enough capacity to cover the largest contingency (the generator producing the most electricity). Within 30 minutes there must be enough capacity to cover the two largest contingencies (the two generators producing the most electricity). Binary variables are used to determine which generator is the largest and the second-largest contingency in each hour. Real variables are used to determine the amount of reserve needed within 10 and 30 minutes. The variables can be seen in table 4.2 below.

Variable	Meaning
$\boldsymbol{U_{i,t}}$	Binary; 1 when generator i is on hour t
$\boldsymbol{S_{i,t}}$	Binary; 1 when generator i is started up in hour t
$\boldsymbol{Shutdown_{i,t}}$	Binary; 1 when generator i is shut down in hour t
$\boldsymbol{cont^1_{i,t}}$	Binary; 1 if generator i is the first contingency in hour t
$\boldsymbol{cont^2_{i,t}}$	Binary; 1 if generator i is the second contingency in hour t
$\boldsymbol{g_{i,t}}$	Amount generated from generator i in hour t
$\boldsymbol{D_{z,t}}$	Demand in each zone in hour t
$\boldsymbol{Contingency^1_t}$	1st contingency amount in hour t
$\boldsymbol{Contingency^2_t}$	2nd contingency amount in hour t
$\boldsymbol{res^{10}_{i,t}}$	10 minute reserves from each generator i in each hour t
$\boldsymbol{res^{30}_{i,t}}$	30 minute reserves from each generator i in each hour t
$\boldsymbol{E_{t,n}}$	Net export from each node n in each hour t

Table 4.2: Definition of variables

4.2 Objective Function

The objective is to maximize social welfare. Welfare is defined as the integral of the inverse demand function ($P_{z,t}(x)$) minus cost. The demand and marginal cost are programmed in the model as step functions.

The objective function stated in mathematical terms is:

$$Max\ \left(\sum_z \sum_t \int_0^{D_{z,t}} P_{z,t}\,(x)dx\right) - \left(\sum_i \sum_t g_{i,t} * c_i^m\right) - \left(\sum_i \sum_t U_{i,t} * c_i^f\right)$$

$$- \left(\sum_i \sum_t s_{i,t} * c_i^{start}\right)$$

4.3 Constraints

A variety of constraints relating to generators, transmission lines and reserve requirements are used to model the system. Below is a list of the constraints used. An explanation of the constraints follows. A capital bold M is represents a very large number, and varies depending on the specific constraint set.

$$mincap_i * U_{i,t} \leq \; g_{i,t} \; \leq \; maxcap_i * U_{i,t} \tag{1}$$

$$-maxdec_i - \; \boldsymbol{M} * \left(1 - U_{i,t}\right) \leq \; g_{i,t} - g_{i,t-1} \tag{2}$$

$$g_{i,t} - g_{i,t-1} \leq -maxinc_i + \; \boldsymbol{M} * \left(1 - U_{i,t-1}\right) \tag{3}$$

$$\left(\sum_s^{[Max\,(1,t-minon_i+1)]} s_{i,s}\right) \leq \; U_{i,t} \tag{4}$$

$$\left(\sum_s^{[Max\,(1,t-minoff_i+1)]} shutdown_{i,s}\right) \leq 1 - \; U_{i,t} \tag{5}$$

$$mustrun_{i,t} \leq U_{i,t} * \boldsymbol{M} \tag{6}$$

$$shutdown_{i,t} \; \geq \; U_{i,t-1} - U_{i,t} \tag{7}$$

$$S_{i,t} \; \geq \; U_{i,t} - U_{i,t-1} \tag{8}$$

$$Contingency_t^1 \geq g_{i,t} \tag{9}$$

$$Contingency_t^1 - g_{i,t} \leq \left(1 - cont_{i,t}^1\right) * \boldsymbol{M} \tag{10}$$

$$\sum_i cont_{i,t}^1 = 1 \tag{11}$$

$$Contingency_t^2 \geq g_{i,t} - cont_{i,t}^1 * \boldsymbol{M} \tag{12}$$

$$Contingency_t^2 - g_{i,t} \leq \left(1 - cont_{i,t}^2\right) * \boldsymbol{M} \tag{13}$$

$$\sum_i cont_{i,t}^2 = 1 \tag{14}$$

$$res_{i,t}^{10} \leq \left(1 - cont_{i,t}^1\right) * maxinc_i/6 \tag{15}$$

$$res_{i,t}^{10} + res_{i,t}^{30} \leq \left(1 - cont_{i,t}^1 - cont_{i,t}^2\right) * maxinc_i/2 \tag{16}$$

$$\left(\sum_i g_{i,t} + res_{i,t}^{10}\right) - \ Contingency_t^1 \geq \sum_z D_{z,t} \tag{17}$$

$$\left(\sum_i g_{i,t} + res_{i,t}^{10} + res_{i,t}^{30}\right) - \ Contingency_t^1 - Contingency_t^2 \geq \sum_z D_{z,t} \tag{18}$$

$$\left(\sum_i g_{i,t} * g_i^l\right) - D_{n,t}^n - E_{t,n} = 0 \tag{19}$$

$$-L_{c,t} \leq \ \sum_n E_{t,n} * ptdf_{c,n} \leq L_{c,t} \tag{20}$$

$$If\ init_i > 0\ \&\ init_i + t \leq minon_i\ then\ U_{i,t}=1 \tag{21}$$

$$If\ init_i < 0\ \&\ t \leq minoff_i + init_i\ then\ U_{i,t}=1 \tag{22}$$

$$If\ s_i^{hour} > 0\ then\ U_{i,s_i^{hour}} = 1 \tag{23}$$

The first constraint ensures that when a generator is on its output is between the minimum and maximum limits. The next two constraints ensure that the ramping limitations are respected. Constraint (2) ensures that a generator does not decrease more than its maximum decrease amount between hours. Constraint (3) is formulated similarly but ensures that a generator does not increase more than its maximum increase amount between hours. The generators must stay on for a minimum amount of time once they have been started up, which is enforced by constraint (4). The generators are also required to stay off for a certain amount of time once they are shutdown, which is enforced by constraint (5). Constraint (6) ensures that generators are forced to be on when they must run, as dictated by the data. Constraint (7) defines a shutdown as occurring when a generator is off in an hour and on in the previous hour. Similarly, constraint (8) defines a startup as occurring when a generator is on in an hour and off in the previous hour.

The next set of constraints enforces the reserve requirements. Constraints (9) and (12) define the size (in MW) of the first and second contingency. Constraint (10) defines the generator that generates $Contingency_t^1$ as the first contingency, while constraint (11) ensures that exactly one generator is the largest contingency. Constraints (13) and (14) similarly define the second contingency. Constraints (15) and (16) limit the amount of 10- and 30-minute reserves that each generator can provide, based on its ramping limit. These constraints also ensure that the generators which are the first and second contingency cannot provide 10- and 30- minute reserves. Constraints (17) and (18) ensure that the 10- and 30-minute reserves provided by all of the generators cover the first and second contingencies. Constraint (19) ensures that the load in each zone is exactly met, either by local generation or by imports. Constraint (20) imposes the limits on power flows along the transmission lines. Constraint (21) and (22) ensures that minimum on time and off time are enforced between days. Constraint (23) forces generators to startup if they were started up on the previous day.

5.0 Data Analysis

The model is programmed in AMPL and solved using cplex. As discussed above, the generators' ramping limits are based off of one day's data. Therefore over-generation and under-generation variables are used to allow these constraints to be relaxed when they must be for feasibility. This only occurs on 8 days: May 14th, May 15th, May 22nd, May 28th, May 29th, May 30th, October 29th, and October 30th. These days are excluded from the data analysis, since the LMPs and other values would be meaningless on these days. The LMP is defined as the dual variable of the load balance constraint (19). It is assumed that the generator is paid for energy based on the LMP. Therefore generator revenue is defined as the LMP corresponding to the location at which the generator is located, multiplied by the amount of energy generated by the generator. Profit is defined as revenue minus cost, where the costs consist of the three components modeled in the objective function. Cases in which a generator has a negative profit for the day, a make-whole payment is made. This is a supplemental payment given to the generator to ensure that no generator operates at a net loss. This make-whole payment mechanism is needed because payments based on energy only can leave a generator with negative profits, due to the non-convex nature of generator startups (Sioshansi et al, 2008).

6.0 Results

The results demonstrate that not solving a unit commitment model to complete optimality can affect LMPs and generator profits. Moreover, these discrepancies do not necessarily balance over the course of the year.

6.1 Generator Profit

All of the near-optimal runs yield differences in generator profits when compared to the optimal run. Table 6.1 shows the average, standard deviation, and coefficient of variation of the difference in annual profits between near- and completely-optimal model runs. An elasticity value of 0 represents case in which the actual 2005 load data is used.

Elasticity	Optimal Compared to	Average	Std Dev	Coefficient of Variation
0	1.E-06	$27,470	$114,522	4.17
0	1.E-04	$47,645	$283,690	5.95
0	1.E-02	$52,319	$216,114	4.13
-0.1	1.E-06	$4,014	$23,736	5.91
-0.1	1.E-04	$12,117	$70,760	5.84
-0.1	1.E-02	$78,462	$290,572	3.70
-0.2	1.E-06	$1,975	$7,499	3.80
-0.2	1.E-04	$16,415	$54,893	3.34
-0.2	1.E-02	$81,990	$304,031	3.71
-0.3	1.E-06	$3,021	$11,071	3.66
-0.3	1.E-04	$20,818	$73,176	3.51
-0.3	1.E-02	$56,590	$209,906	3.71

Table 6.1: Overview Statistics of yearly generator profits

The unit commitment solutions yield many generators that have 0 profits for the entire year (this either occurs because a generator is never started up or because it exactly recovers its costs through energy or make-whole payments). Table 6.2 presents the same summary statistics as table 6.1, with generators that receive zero profit in the complete optimum removed from the sample. Comparing the two tables shows that the profit differences tend to be higher and more variable when these generators are removed from the sample.

Elasticity	Optimal Compared to	Average	Std Dev	Coefficient of Variation	# of gen. included
0	1.E-06	$53,771	$156,006	2.90	141
0	1.E-04	$93,263	$392,175	4.21	
0	1.E-02	$102,412	$294,237	2.87	
-0.1	1.E-06	$8,864	$34,729	3.92	125
-0.1	1.E-04	$26,754	$103,486	3.87	
-0.1	1.E-02	$173,245	$413,153	2.38	
-0.2	1.E-06	$4,543	$10,871	2.39	120
-0.2	1.E-04	$37,755	$78,428	2.08	
-0.2	1.E-02	$188,577	$439,706	2.33	
-0.3	1.E-06	$7,007	$16,046	2.29	119
-0.3	1.E-04	$48,285	$105,554	2.19	
-0.3	1.E-02	$131,252	$304,632	2.32	

Table 6.2: Overview Statistics of yearly generator profits with generators seeing zero profit for the entire year removed.

Figure 6.1 shows the distribution of annual profit differences between the optimal and 1e-4 case with demand elasticity of 0. The figure shows the distribution of the profit differences is unimodal and centered around 0, although there are significant outliers in the data as well.

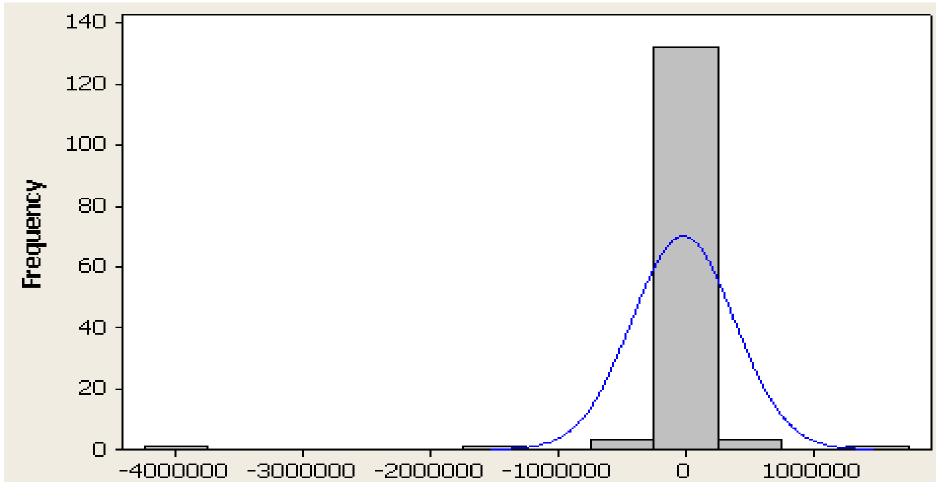

Figure 6.1: Histogram of yearly generator profit difference for the case of 0 elasticity and optimal compared to 1e-4 with the generators seeing 0 profit for the year removed.

Figure 6.2 further demonstrates the extent of these differences, by showing the difference in generator profits as a percentage of profits in the complete-optimal solution.

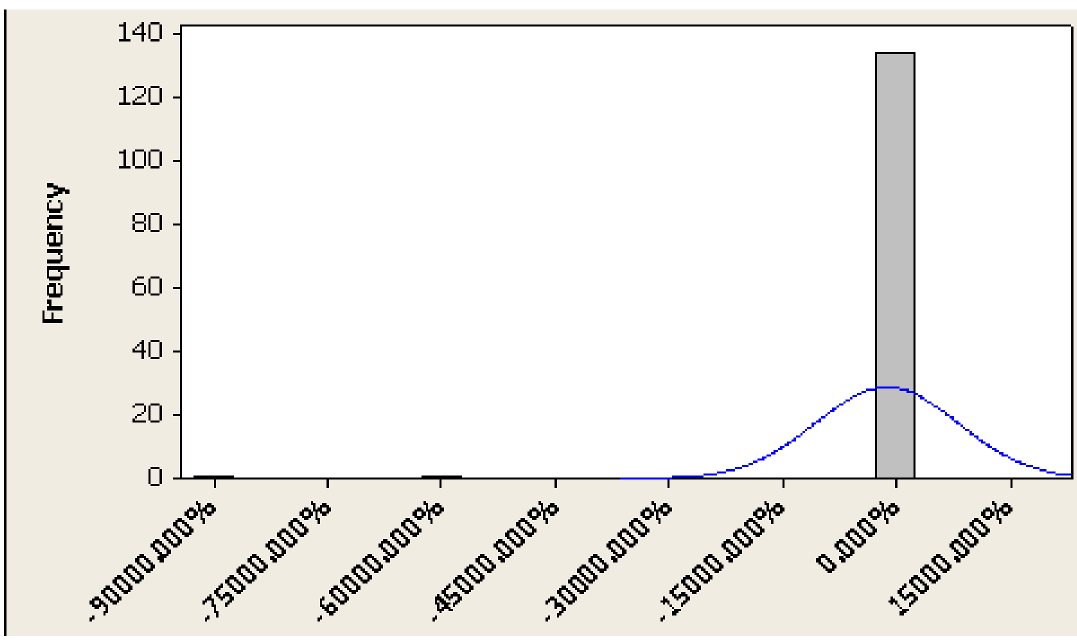

Figure 6.2: Histogram of percent difference in yearly generator profit difference for the case of 0 elasticity and optimal compared to 1e-4 with the generators seeing 0 profit for the year removed.

As can be seen in the histogram the outliers have up to a 90,000% difference in profit, which is a significant profit difference. While this extreme of a profit difference was not witnessed in all cases a large difference was still seen. Figure 6.3 shows a histogram of the percent differences in generator profits between the complete-optimal and a near-optimal solution with an optimality gap of 1e-2, when the demand elasticity is -0.1. This figure does not show as extreme profit differences, but still shows outliers with as high as 500% difference in profit. Outliers like this are more common amongst the other cases. This case again demonstrates a unimodal distribution centered at 0 with extreme outliers.

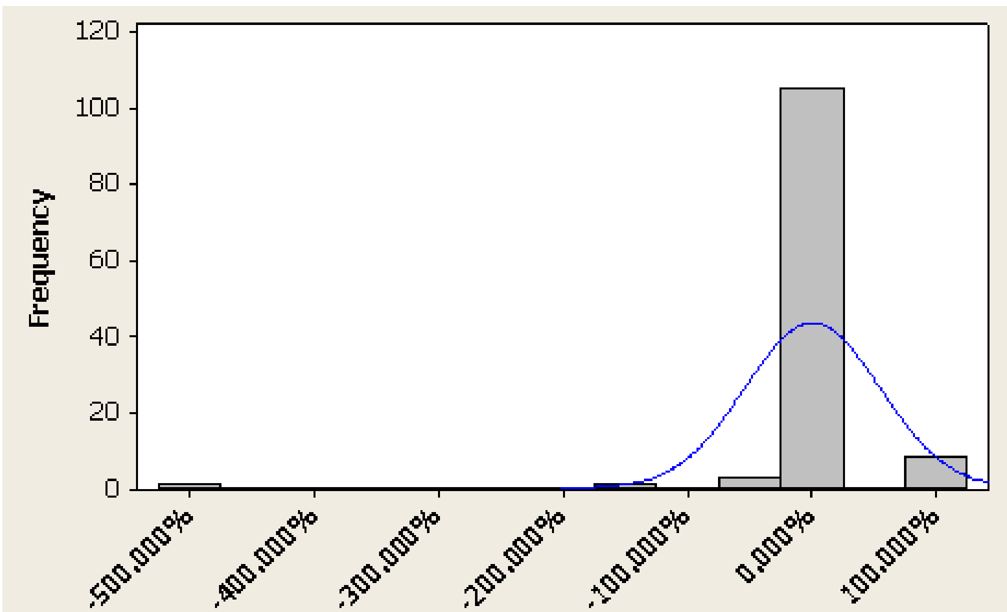

Figure 6.3: Histogram of percent difference in yearly generator profit difference for the case of -0.1 elasticity and optimal compared to 1e-2 with the generators seeing 0 profit for the year removed.

Generators which have an annual profit difference greater than $100,000 for the case of elasticity of 0 and optimal compared to 1e-4 are examined more closely. Table 6.3 shows the annual profit differences for these generators. These results demonstrate that as the model is solved closer to optimal the profit difference for individual generators is not necessarily decreased. The table shows the values of near-optimal subtracted from optimal, so a negative value means that the profit under the near-optimal solution is higher.

Elasticity	0	0	0	1	1	1	2	2	2	3	3	3
Optimal -	**1.E-06**	**1.E-04**	**1.E-02**	**1.E-06**	**1.E-04**	**1.E-02**	**1.E-06**	**1.E-04**	**1.E-02**	**1.E-06**	**1.E-04**	**1.E-02**
UNIT004	$92,061	-$314,670	$54,843	-$40,010	-$246,156	-$580,674	$19,591	$93,730	$249,507	$0	$0	$0
UNIT070	$456,183	$508,038	$1,023,380	-$6,144	$28,116	$455,237	-$11,886	$84,191	$707,015	-$27,107	$126,975	-$78,401
UNIT076	$529,127	$443,482	$580,435	-$107,926	-$132,942	$857,253	$12,341	$282,771	$1,597,265	$125,670	$549,308	$1,305,746
UNIT077	$1,319,345	$1,319,564	$1,967,877	-$84,615	$91,135	$1,650,164	-$34,982	$212,024	$2,042,427	-$34,589	$230,219	$765,645
UNIT078	$260,394	$150,064	$140,460	-$36,473	-$85,167	$1,111,040	$22,343	$284,683	$1,283,779	-$402	$2,673	$8,890
UNIT088	-$68,781	-$731,078	-$141,422	-$52,583	-$228,940	-$119,831	$74,900	$448	$79,220	$0	$0	$0
UNIT090	$200,325	$221,374	$375,956	-$15,012	$43,815	$2,950	-$5,295	-$7,885	$67,463	$49,036	$97,719	$49,826
UNIT091	$237,836	$246,496	$412,725	-$13,433	$38,935	-$51,433	-$1,718	-$19,952	$51,371	-$828	$5,513	$18,335
UNIT153	$422,814	$223,705	$162,018	-$54,429	-$144,426	$1,834,767	$38,312	$485,448	$2,160,114	$2,394	$10,463	$24,871
UNIT224	-$6,744	-$314,497	-$132,302	$8,610	$530	-$52,744	$7,530	-$17,133	-$54,782	-$7,575	$510,202	$423,508
UNIT225	-$893,676	-$1,274,753	-$716,260	-$9,092	-$21,712	$255,663	$6,517	$77,627	$352,234	-$10,058	$92,513	$286,916
UNIT227	$195,502	$120,692	$347,802	-$21,061	-$67,578	$1,008,599	$21,693	$242,312	$1,192,379	$0	$0	$0
UNIT235	$4,510	-$223,355	-$1,333,361	$4,299	-$180,729	-$3,024,998	-$5,161	-$141,922	-$2,411,777	-$435	-$9,661	-$70,455
UNIT285	-$459,658	-$4,209,027	$1,929,801	$356,401	$1,073,972	$1,208,961	-$62,777	$406,339	$1,851,787	$0	$0	$0
UNIT329	-$40,165	-$159,010	-$33,101	-$415	$225	$225	$0	$0	$0	$0	$0	$0
UNIT375	$194,256	$111,949	$104,784	-$27,209	-$63,535	$828,845	$16,668	$212,376	$957,711	$0	$0	$0
UNIT377	-$28,800	$287,026	$1,084,672	-$25,505	$26,021	$595,857	-$10,146	$59,937	$595,581	$0	$0	$0
UNIT379	$96,252	$150,858	$150,858	$0	$0	$0	$0	$0	$0	$0	$0	$0

Table 6.3: Yearly Profit Difference between optimal and near optimal runs at varying elasticity values for generators with the largest difference.

For example, unit 285 has a $2 million profit difference when the model has an optimality gap of 1e-2. If the optimality gap is reduced to 1e-4, this profit difference decreases to -$4 million. This gives a $6 million swing in profits between these two sets of near-optimal solutions. There are many cases in the table where a generators profit difference fluctuates greatly and without trend between cases and this occurs at all elasticity values. The lack of pattern in generator profit difference between runs demonstrates that solving closer to optimal does not alleviate this issue, unless the problem can be solved to complete optimality.

Of the 18 generators with a profit difference of $100,000 or more, 12 generators had a daily or almost daily profit difference throughout the year. Figure 6.4 shows the daily profit difference for a subset of the generators with the highest profit differences. This figure shows that large profit difference in most cases is caused by a daily profit difference; however there are some cases where the profit difference occurs on only a few days.

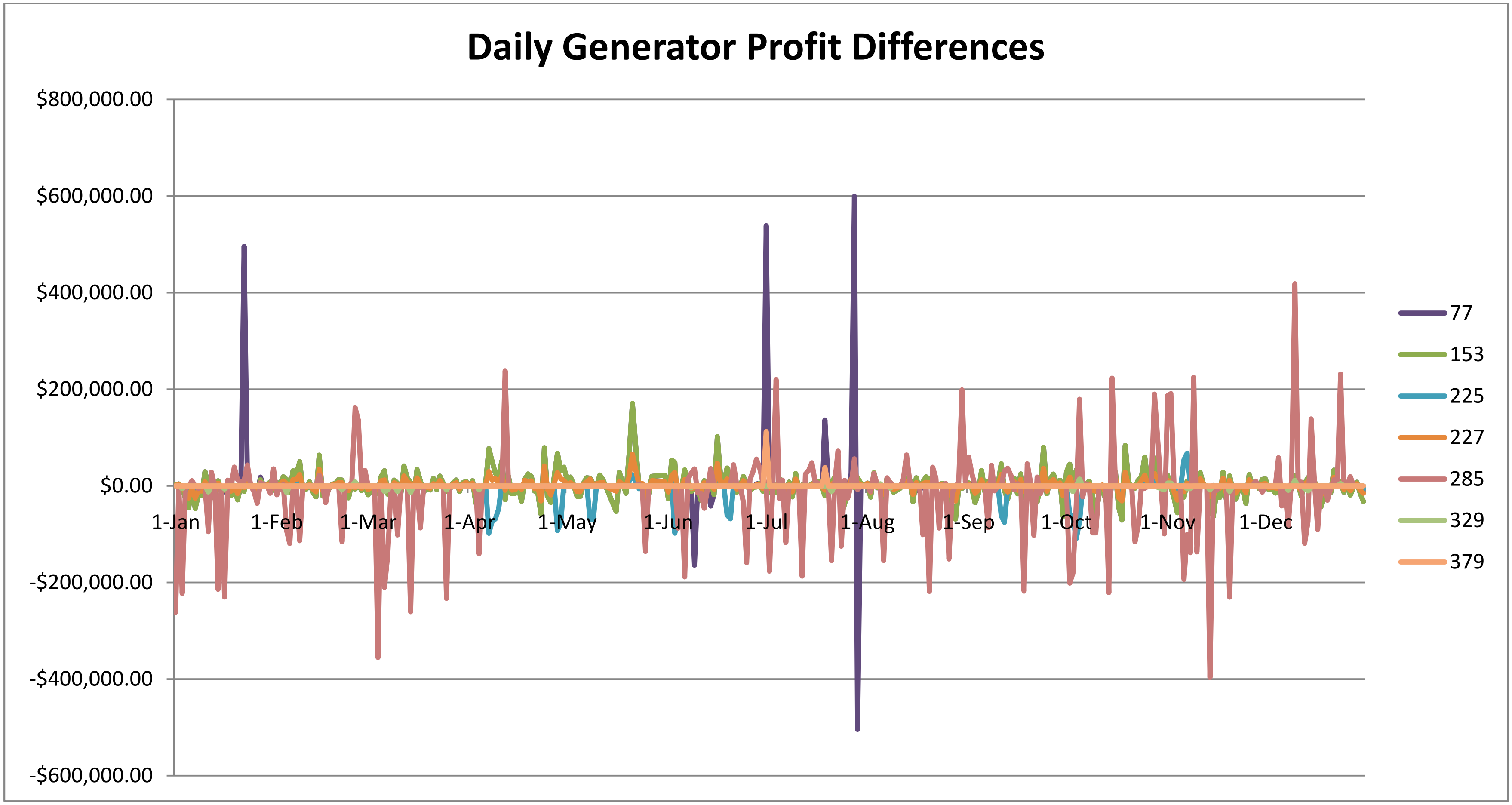

Figure 6.4: Yearly Profit Difference between optimal and 1e-4 run at 0 elasticity values for generators with the large difference.

The correlation between the different near-optimal runs was computed to statistically show that there is no relation between the results, demonstrating that solving closer to optimal does not reduce individual generators' profit differences. Figure 6.5 is a scatterplot of the difference between optimal and 1e-6 and 1e-2 in generator yearly profits of an elasticity value of -0.2. The difference between optimal and 1e-2 is on the x-axis and the difference between optimal and 1e-6 is on the y-axis. It is clear from the scatterplot that there is no relation between the runs and the regression analysis further demonstrates this.

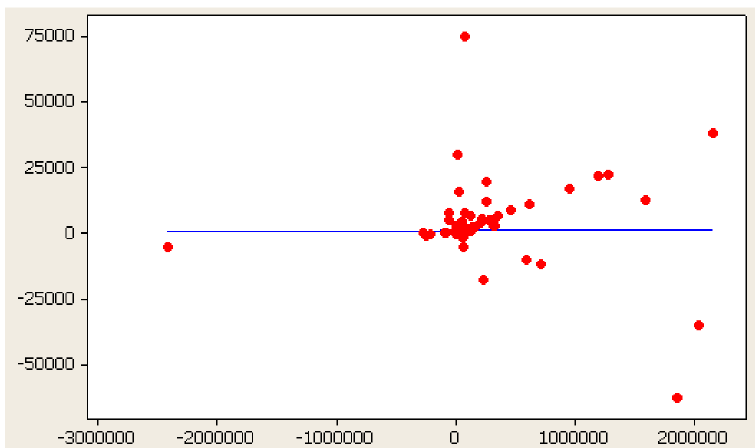

Figure 6.5: Comparison of difference between optimal and 1e-6 and 1e-2 in generator yearly profits for elasticity value of -0.2.

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Table 6.4 gives the R^2 and P value of these regressions, which further demonstrates that there is no correlation.

Elasticity	R^2	P value
0	24.2%	0.000
-0.1	0.0%	0.900
-0.2	0.0%	0.922
-0.3	0.0%	0.739

Table 6.4: R-square and P values for regression analysis of generator profit differences of optimal compared to 1e-6 and 1e-2 cases.

6.2 Generator Profits explanation of results

The results clearly show that certain generators saw larger profit differences between optimal and near-optimal runs than others. Table 6.5 gives some general characteristics of the generators with the largest profit differences. The table shows that there is no over-arching trend to explain why these generators saw larger profit difference.

Unit Name	Min On	Min Off	Max Dec	Max Inc	Fixed Cost	Startup Cost
UNIT004	24	24	300	300	$9,600.00	$53,333.33
UNIT070	13	7	1440	1440	$0.00	$30,000.00
UNIT076	24	48	180	180	$0.00	$0.00
UNIT077	24	48	240	240	$0.00	$0.00
UNIT078	1	1	300	300	$0.00	$0.00
UNIT088	8	5	300	300	$0.00	$12,671.33
UNIT090	14	6	420	420	$0.00	$21,258.53
UNIT091	14	6	420	420	$0.00	$21,255.67
UNIT153	24	49	60	60	$0.00	$0.00
UNIT224	1	1	1500	1500	$0.00	$1,000.00
UNIT225	8	36	150	150	$0.00	$21,794.67
UNIT227	24	48	300	300	$0.00	$60,166.24
UNIT235	1	1	1500	1500	$0.00	$0.00
UNIT285	1	1	3600	3600	$0.00	$600.00
UNIT329	1	1	120	120	$582.79	$861.62
UNIT375	1	1	300	300	$0.00	$0.00
UNIT377	24	8	180	180	$1,423.00	$108,168.00
UNIT379	14	8	300	300	$3,445.36	$36,657.27

Table 6.5: Minimum on time, minimum off time, ramping rates, fixed cost and startup costs for generators which saw large profit difference.

While an overall trend in why these generators se a large profit difference does not exist, an explanation for their profit difference can be found when looking at them individually. For example Unit 77 has very low costs but must remain on for 24 hours once started. This unit also has very low ramping rates. Therefore a decision to turn on this generator or increase or decrease its generation has a significant effect on its production in successive hours. Thus one different commitment or dispatch decision between the different cases can affect this generator's profits in many subsequent hours.

Generator 285, on the other hand, must only be on or off for one hour at a time. This type of versatility allows many differences between solutions in which the generator is started up and turned off. Generator 379, on the other hand, only has differences in its commitment and dispatch between the optimal and near optimal solution on a few days. This generator has a high start-up cost so on most days when it is used, it receives a make-whole payment. However in the optimal solution there are two days during which it stays on long enough to make profit whereas in the near-optimal solutions it has no profit. These profit differences show that the prices generated by the market can send spurious signals regarding what types of generation investors should build.

6.3 Locational Marginal Price

The difference in the LMP between the near-optimal and optimal runs is very small in most hours. However there are outliers in which the price difference is very significant. Table 6.6 shows the average differences between completely and near-optimal model runs.

		Average			Std Dev		
	Optimal Compared to	**1.E-02**	**1.E-04**	**1.E-06**	**1.E-02**	**1.E-04**	**1.E-06**
Elasticity	**Zone**						
0	CONNECTICUT	-0.20	-0.14	-0.13	8.87	9.98	9.42
0	MAINE	-0.02	-0.03	-0.04	3.23	2.76	2.75
0	NEMASS	0.00	-0.02	-0.04	3.31	2.82	2.79
0	NEWHAMPSHIRE	-0.02	-0.03	-0.04	3.23	2.76	2.75
0	RHODEISLAND	-0.02	-0.03	-0.04	3.23	2.76	2.75
0	SEMASS	-0.02	-0.03	-0.04	3.23	2.76	2.75
0	VERMONT	-0.02	-0.03	-0.04	3.23	2.76	2.75
0	WCMASS	-0.02	-0.03	-0.04	3.23	2.76	2.75
-0.1	CONNECTICUT	0.06	0.02	0.03	4.25	2.47	1.28
-0.1	MAINE	-0.19	0.01	0.01	1.61	1.15	0.55
-0.1	NEMASS	-0.17	0.01	0.00	1.73	1.25	0.62
-0.1	NEWHAMPSHIRE	-0.19	0.01	0.01	1.61	1.15	0.55
-0.1	RHODEISLAND	-0.19	0.01	0.01	1.61	1.15	0.55
-0.1	SEMASS	-0.19	0.01	0.01	1.61	1.15	0.55
-0.1	VERMONT	-0.19	0.01	0.01	1.61	1.15	0.55
-0.1	WCMASS	-0.19	0.01	0.01	1.61	1.15	0.55
-0.2	CONNECTICUT	-0.20	-0.01	0.00	1.98	1.42	0.53
-0.2	MAINE	-0.22	-0.05	0.00	1.59	1.13	0.45
-0.2	NEMASS	-0.22	-0.05	0.00	1.62	1.15	0.45
-0.2	NEWHAMPSHIRE	-0.22	-0.05	0.00	1.59	1.13	0.45
-0.2	RHODEISLAND	-0.22	-0.05	0.00	1.59	1.13	0.45
-0.2	SEMASS	-0.22	-0.05	0.00	1.59	1.13	0.45
-0.2	VERMONT	-0.22	-0.05	0.00	1.59	1.13	0.45
-0.2	WCMASS	-0.22	-0.05	0.00	1.59	1.13	0.45
-0.3	CONNECTICUT	-0.13	-0.05	-0.01	1.87	1.48	0.96
-0.3	MAINE	-0.13	-0.04	0.01	1.38	1.06	0.47
-0.3	NEMASS	-0.12	-0.04	0.01	1.47	1.15	0.65
-0.3	NEWHAMPSHIRE	-0.13	-0.04	0.01	1.38	1.06	0.47
-0.3	RHODEISLAND	-0.13	-0.04	0.01	1.38	1.06	0.47
-0.3	SEMASS	-0.13	-0.04	0.01	1.38	1.06	0.47
-0.3	VERMONT	-0.13	-0.04	0.01	1.38	1.06	0.47
-0.3	WCMASS	-0.13	-0.04	0.01	1.38	1.06	0.47

Table 6.6: Yearly average price difference (in $/ megawatt hour) between optimal and near optimal run in each zone for all elasticity cases.

The table shows that the price differences average to around twenty cents. This is small compared to the average hourly price for the optimal case of $43.95. As previously stated, while the average price differences are small there are still cases of significant price differences between optimal and near-optimal cases. Figure 6.6 shows hours in which the LMP has a difference of $10/MWh. This is for the case of no demand elasticity and the optimal solution compared to an optimality gap of 1e-4.

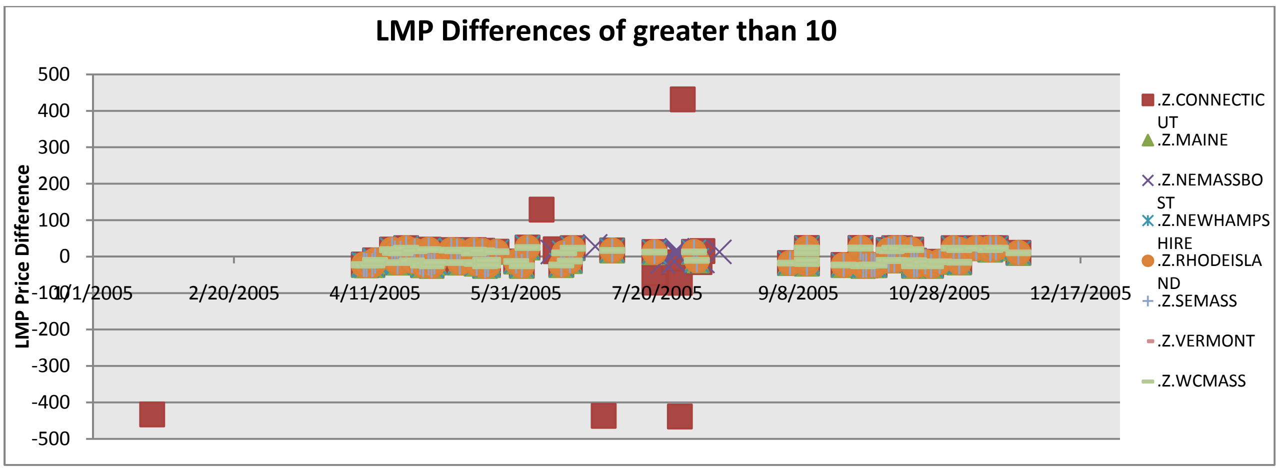

Figure 6.6: LMP difference between 1e-4 and optimal of greater than 10 for the 0 elasticity case.

Most of the cases shown in figure 6.7 have a price difference of less than $50/MWh; however there are cases in which the price difference is over $400. These extreme price differences occur in the Connecticut zone. Figure 6.7 shows absolute LMP differences greater than $10/MWh, by time of day.

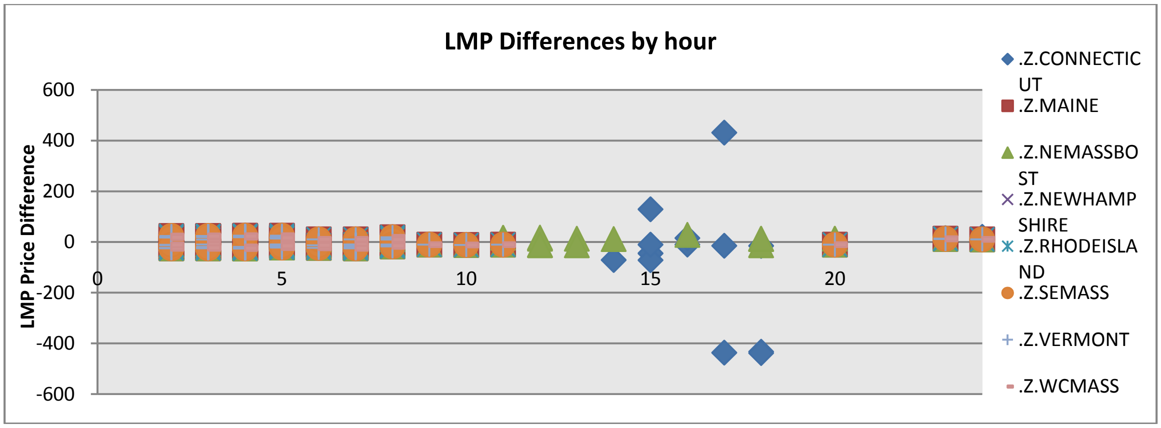

Figure 6.7: LMP difference between 1e-4 and optimal of greater than 10 for the 0 elasticity case plotted by hour in the day.

As can be seen from the figure above large price differences were most common in the afternoon between 3 and 6 pm. Smaller price differences occurred in the night hours but at a higher frequency throughout the year. A similar analysis does not reveal a strong seasonal trend. The most significant trend in price differences seem to be locational, with the Connecticut and Boston zones being more prone to large deviations than the other network locations. There are also many days on which all of the zones see the same price difference.

6.4 Make-Whole Payment

As previously stated the make-whole payment is a supplemental payment made to generators at the end of each day on which they have a negative profit. This payment is exactly equal to the revenue shortfall, and brings their profit to zero. There is a small difference in the total amount of make-whole payments made between all of the runs completed. Table 6.7 summarizes the total annual make-whole payments and the average per-unit payment.

Elasticity	Solved to	Sum	Average per unit
0	**Optimal**	$3,075,027,762	$11,141,405
0	**1.E-06**	$3,077,034,833	$11,148,677
0	**1.E-04**	$3,064,100,911	$11,101,815
0	**1.E-02**	$3,075,231,486	$11,142,143
-0.1	**Optimal**	$2,990,356,132	$10,834,624
-0.1	**1.E-06**	$2,990,091,730	$10,833,666
-0.1	**1.E-04**	$2,987,887,030	$10,825,678
-0.1	**1.E-02**	$2,977,053,678	$10,786,426
-0.2	**Optimal**	$2,951,801,037	$10,694,931
-0.2	**1.E-06**	$2,951,963,723	$10,695,521
-0.2	**1.E-04**	$2,952,985,831	$10,699,224
-0.2	**1.E-02**	$2,947,604,035	$10,679,725
-0.3	**Optimal**	$2,923,643,845	$10,592,912
-0.3	**1.E-06**	$2,923,770,412	$10,593,371
-0.3	**1.E-04**	$2,924,989,712	$10,597,789
-0.3	**1.E-02**	$2,918,977,863	$10,576,007

Table 6.7: Make whole payment amount for each of the runs completed.

7.0 Conclusions

Examining yearly generator profits for each unit shows that generator profit differences between optimal and near-optimal runs do not balance over a year. Generator profit differences often occur on a daily basis. Solving closer to optimal does not necessarily decrease differences in individual generator profits. Therefore if the model is not able to be solved to complete optimality it will suffer equity issues. This can have significant effect on price signals. Particularly it can affect the information investors are using to make decisions. Examining the locational marginal prices in eight zones showed variance in price also occurs when not solved to optimal.

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